Analysis 2

30 April 2024

Warm-up: Is $y = \frac{1}{3}x^3 - 4x$ concave up, concave down, or neither at $x = 2$?

Seeing f′ and f″ in graphs

Last time

Monotonicity

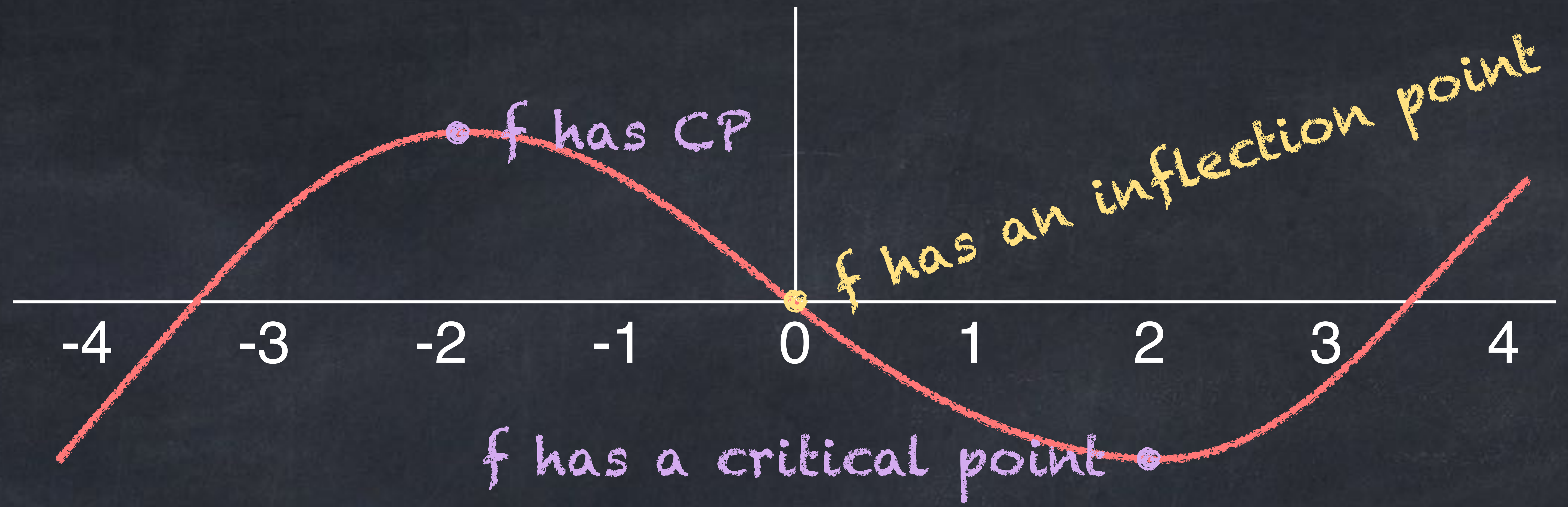
- If $f' > 0$ then f is increasing,
- If $f' < 0$ then f is decreasing.
- A **critical point** is an x-value* where f' *is zero* or *doesn't exist*.

Concavity

- If $f'' > 0$ then f is concave up,
- If $f'' < 0$ then f is concave down.
- An **inflection point** is an x-value* where f'' *changes sign*.

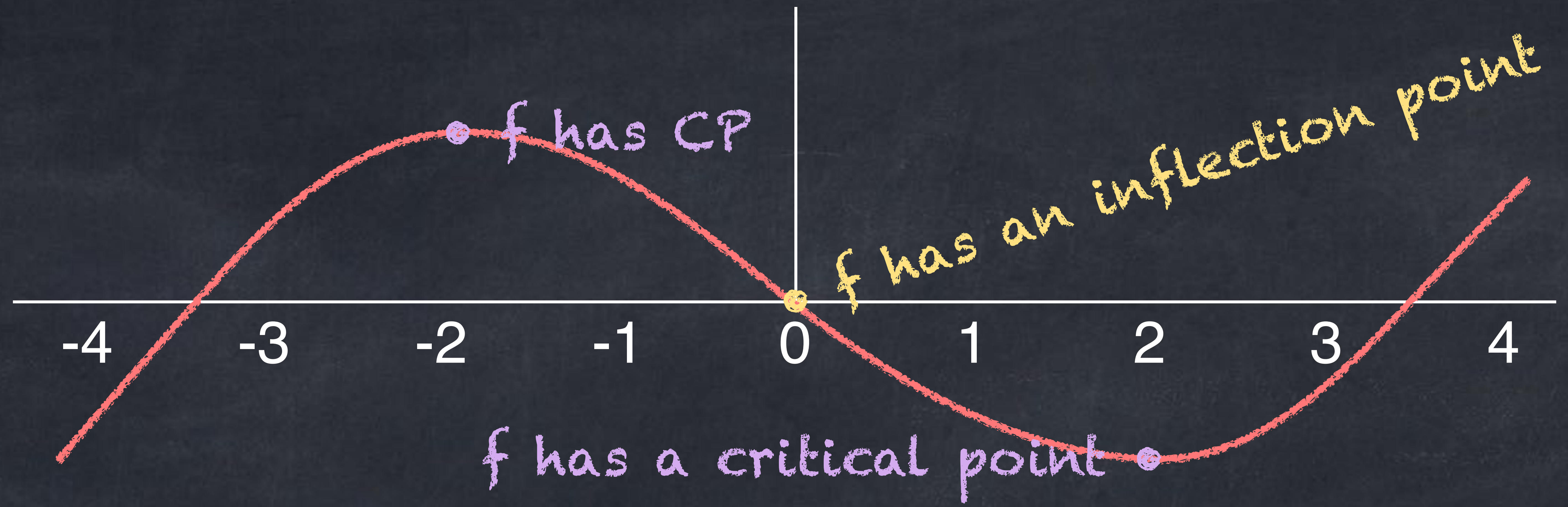
* The x-value must to be in the domain of f.

For $f(x) = \frac{1}{3}x^3 - 4x$,



<table>
<tr><td></td><td>f ⛔</td><td colspan="2">f ➕</td><td colspan="2">f ⛔</td><td>f ➕</td></tr>
<tr><td>f′ tells us</td><td colspan="2">f ↗️</td><td colspan="2">f ↘️</td><td colspan="2">f ↗️</td></tr>
<tr><td>f″ tells us</td><td colspan="3">f ☹️</td><td colspan="3">f 🙂</td></tr>
</table>

For $f(x) = \frac{1}{3}x^3 - 4x$,



<table>
<tr><td></td><td>f is negative</td><td colspan="2">f is positive</td><td colspan="2">f is negative</td><td>f is pos.</td></tr>
<tr><td>f′ tells us</td><td colspan="2">f is increasing
f′ is positive</td><td colspan="2">f is decreasing
f′ is negative</td><td colspan="2">f is increasing
f′ is positive</td></tr>
<tr><td>f″ tells us</td><td colspan="3">f is concave down
f′ is decreasing
f″ is negative</td><td colspan="3">f is concave up
f′ is increasing
f″ is positive</td></tr>
</table>

Task: Find (a) the critical points and (b) the inflection points of

$$f(x) = \frac{3}{20}x^5 - x^4 + 2x^3 - 5x + 7.$$

Possible hints:

$$\begin{aligned}
\frac{3}{4}x^4 - 4x^3 + 6x^2 - 5 &= \frac{1}{4}(x^2 - 2x - 2)(3x^2 - 10x + 10) \\
3x^3 - 12x^2 + 12x &= 3x(x^2 - 4x + 4) \\
9x^2 - 24x + 12 &= 3(x - 2)(3x - 2) \\
18x - 24 &= 6(3x - 4)
\end{aligned}$$

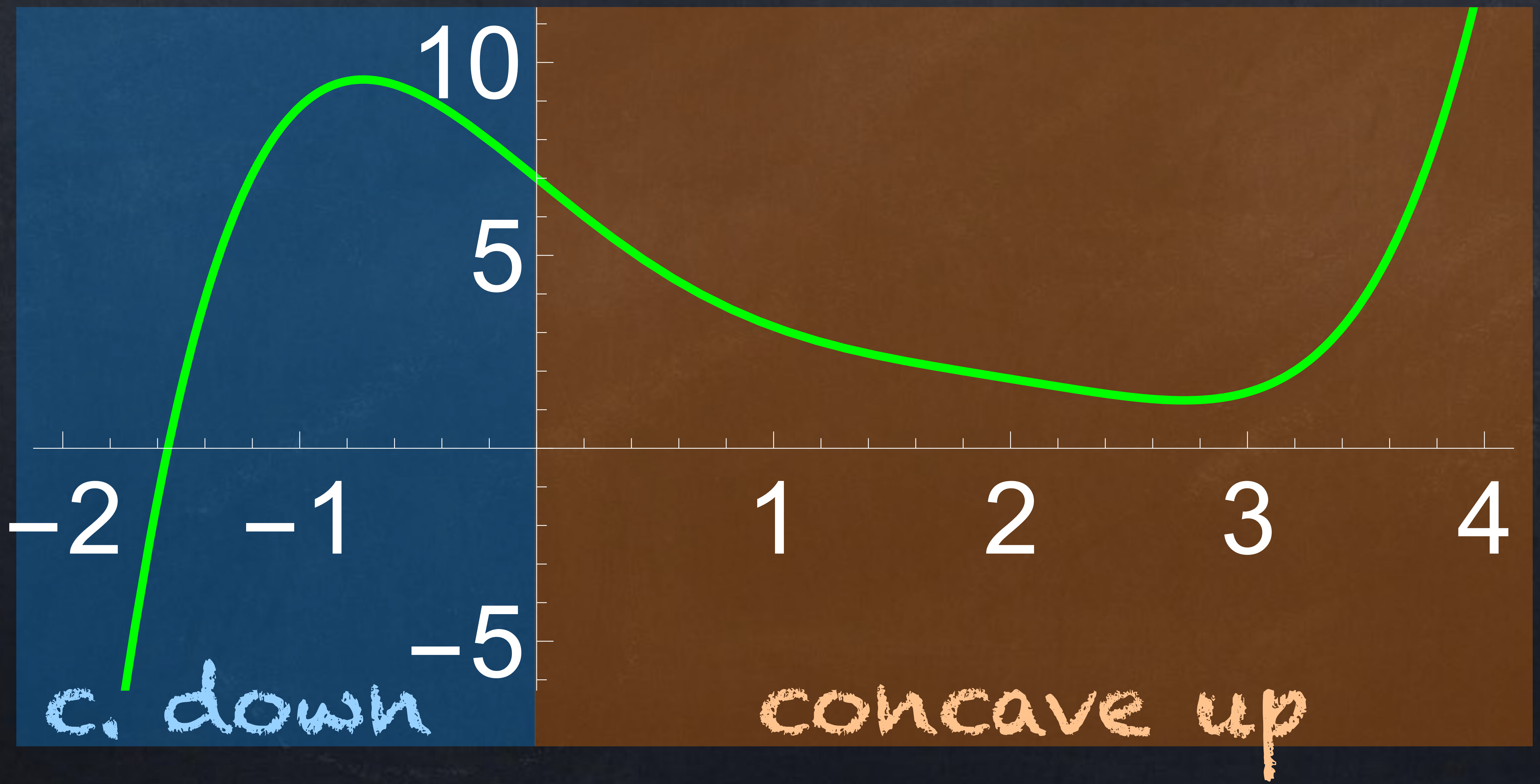
Answer: critical points: $x = 1-\sqrt{3}$ and $x = 1+\sqrt{3}$

infection points: $x = 0$ only

Task: Find (a) the critical points and (b) the inflection points of

$$f(x) = \frac{3}{20}x^5 - x^4 + 2x^3 - 5x + 7.$$

Although $f''(2) = 0$, there is no inflection point there (f'' does not change sign).



Celebration of Knowledge 🥳

Exam 1 is next week.

Topics:

- calculating derivatives
  - Power Rule, Product Rule, Chain Rule, etc.
- tangent lines
- monotonicity (increasing vs. decreasing) and critical points
- concavity (concave up vs. concave down) and infection points
- extrema (minima and maxima)

Exam Review

Task 1: Find the slope of the tangent line to $y = \cos(8x)$ at $x = \frac{\pi}{3}$.

This is $f'(\pi/3)$.

(No ... = 0. No ... > 0. Just plug $\pi/3$ into derivative and stop.)

Task 2: Give an equation for the tangent line to $y = \cos(8x)$ at $x = \frac{\pi}{3}$.

The format $y - y_0 = m(x - x_0)$ is easier than $y = ax + b$.

Task 3: Give the derivative of $f(x) = x^3\sqrt{9 + \sin\left(x^2\right)}$ as a function of x.

Start with the Product Rule.

We will also need the Chain Rule twice (and Sum once).